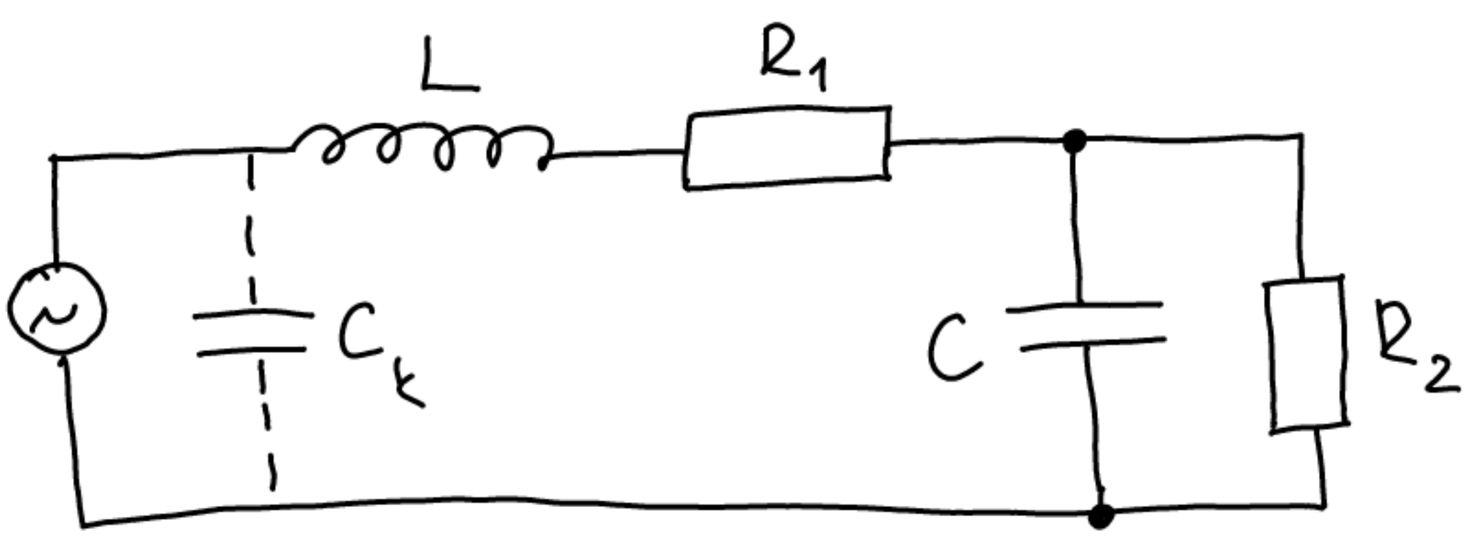

$R_1 = 500\ \Omega$

$R_2 = 1.5\ k\Omega = 1500\ \Omega$

$L = 420\ mH = 0.42\ H$

$C = 210\ nF = 210 \cdot 10^{-9}\ F$

$u(t) = 35.35 \cdot \sin(\omega \cdot t + 1.234)\ V$

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a) <u>I.a.</u> $\underline{U} = \frac{35.35}{\sqrt{2}} \angle 1.234\ rad = \boxed{25.00 \angle 70.7^\circ = 8.263 + 23.595 \cdot i}$

<u>I.b.</u> $f = 500\ Hz \rightarrow \omega = 2 \cdot \pi \cdot f = \boxed{3142\ \frac{rad}{s}}$

<u>I.c.</u> $\underline{Z}_L = \omega \cdot L \angle 90^\circ = \boxed{1320 \angle 90^\circ\ \Omega = 0 + 1320 \cdot i}$

$\underline{Z}_C = \frac{1}{\omega \cdot C} \angle -90^\circ = \boxed{1516 \angle -90^\circ\ \Omega = 0 - 1516 \cdot i}$

<u>II.a.</u> R_1 i L su spojeni SERIJSKI

$\underline{Z}_1 = R_1 + \underline{Z}_L = \boxed{500 + 1320 \cdot i = 1412 \angle 69.3^\circ\ \Omega}$

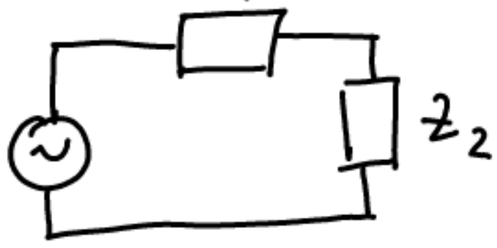
<u>II.b.</u> R_2 i C su spojeni PARALELNO

$\underline{Z}_2 = \frac{R_2 \cdot \underline{Z}_C}{R_2 + \underline{Z}_C} = \frac{(1500 \cdot 1516) \angle 0^\circ - 90^\circ}{1500 - 1516 \cdot i} = \frac{(1500 \cdot 1516) \angle -90^\circ}{2133 \angle -45.3^\circ} =$

$$\boxed{Z_2 = 1066 \angle -44.7^\circ\ \Omega = 757.7 - 749.8 \cdot i}$$

II.c. Zato što je to mješoviti spoj i treba ga se svesti na serijski ili paralelni da bi se mogla izračunati ukupna impedancija.

II.d. Spoj Z_1 i Z_2 je SERIJSKI.



$$Z_{uk} = Z_1 + Z_2 = (500 + 1320i) + (757.7 - 749.8 \cdot i) =$$

$$\boxed{= 1257.7 + 570.2 \cdot i = 1381 \angle 24.4^\circ\ \Omega}$$

III.a. $$I_{uk} = \frac{U}{Z_{uk}} = \frac{25 \angle 70.7^\circ}{1381 \angle 24.4^\circ} = \boxed{18.10 \angle 46.3^\circ\ mA}$$

$$\boxed{i_{uk}(t) = 25.60 \cdot \sin(3142 \cdot t + 0.808)\ mA}$$

III.b. Za izračunati I_c i I_{R_2} potreban je napon U_{Z_2} koji je isti u obje grane: $U_{Z_2} = U_{R_2} = U_c$

III.c. Napon se od izvora do čvora „troši" na R_1 i L.

$$U_L = I_{uk} \cdot Z_L = (18.1 \cdot 10^{-3} \cdot 1320) \angle 46.3^\circ + 90^\circ = 23.89 \angle 136.3^\circ\ V$$

$$\boxed{= 23.89 \angle 136.3^\circ\ V = -17.27 + 16.51 \cdot i}$$

$$U_{R_2} = I_{uk} \cdot R_2 = (18.1 \cdot 10^{-3} \cdot 500) \angle 46.3^\circ + 0^\circ =$$

$$\boxed{= 9.05 \angle 46.3^\circ \; V = 6.252 + 6.543 \cdot i}$$

III.d. $U_{RC} = U - U_L - U_{R_1} =$

$$= (8.263 + 23.595\,i) - (-17.27 + 16.51\,i) - (6.252 + 6.543\,i) =$$

$$\boxed{= 19.281 + 0.542 \cdot i = 19.29 \angle 1.6^\circ \; V}$$

III.e.

$$U_{RC} = I_{uk} \cdot Z_2 = (18.1 \cdot 10^{-3} \cdot 1066) \angle 46.3^\circ - 44.7^\circ =$$

$$= 19.29 \angle 1.6^\circ \checkmark$$

III.f. $I_c = \dfrac{U_{RC}}{Z_c} = \dfrac{19.29 \angle 1.6^\circ}{1516 \angle -90^\circ} = \boxed{12.72 \angle 91.6^\circ \; mA}$

$$I_{R_2} = \frac{U_{RC}}{R_2} = \frac{19.29 \angle 1.6^\circ}{1500 \angle 0^\circ} = \boxed{12.86 \angle 1.6^\circ \; mA}$$

IV.a. $S_{uk} = U \cdot I_{uk}^* = 25 \angle 70.7^\circ \cdot 18.1 \cdot 10^{-3} \angle -46.3^\circ =$

$$\boxed{= 452.5 \angle 24.4^\circ \; mVA}$$

kut mora biti isti kao od Z_{uk}! (i je)

IV.b. $S_{uk} = \left(452.5 \cdot \cos(24.4°) + 452.5 \cdot \sin(24.4°)\,i\right) mVA =$

$= 412.1 + 186.9 \cdot i$

$\Rightarrow$ **$P_{uk} = 412.1\ mW$**

$Q_{uk} = 186.9\ mVAr$

IV.c. $S_{uk} = U \cdot I_{uk}^{*} = U \cdot \frac{U^{*}}{Z_{uk}^{*}} = \frac{|U|^2}{Z_{uk}^{*}} = \frac{25^2}{1381\angle -24.4°} = 0.4526\angle 24.4°$

↑ tu je jedina razlika, to je zbog zaokruživanja međurezultata

IV.d. $S_{uk} = U \cdot I_{uk}^{*} = I_{uk} \cdot Z_{uk} \cdot I_{uk}^{*} = |I|^2 \cdot Z_{uk} =$

$= \left(18.1 \cdot 10^{-3}\right)^2 \cdot 1381\angle 24.4° =$

$= 0.4524\angle 24.4°$

IV.e. $P_{R_1} = |I_{uk}|^2 \cdot R_1 = 18.1^2 \cdot 500 \cdot 10^{-6}\angle 0° =$ **$163.8\ mW$**

$P_{R_2} = |I_{R_2}|^2 \cdot R_2 = 12.86^2 \cdot 1500 \cdot 10^{-6}\angle 0° =$ **$248.1\ mW$**

provjera: $P_{R_1} + P_{R_2} = 411.9 \approx P_{uk}$ ✓

$Q_L = |I_{uk}|^2 \cdot Z_L = 18.1^2 \cdot 1320 \cdot 10^{-6}\angle +90° =$ **$+432.4\ mVAr$**

$Q_c = |I_c|^2 \cdot Z_c = 12.72^2 \cdot 1516 \cdot 10^{-6}\angle -90° =$ **$-245.3\ mVAr$**

provjera: $Q_L + Q_c = 187.1 \approx Q_{uk}$ ✓

b) I. Napon će biti isti kao napon izvora, jer je spojen paralelno na izvor.

II. $Z_{C_k} = \frac{1}{\omega \cdot C_k} \angle -90°$

III. $Q_{C_k} = U \cdot I_{C_k}^* = \frac{U \cdot U^*}{Z_{C_k}^*} = \frac{|U|^2}{Z_{C_k}^*} = \frac{|U|^2}{\frac{1}{\omega \cdot C_k} \angle +90°} = \omega \cdot C_k \cdot |U|^2 \angle -90°$

IV. Zato što je napon prema ostatku strujnog kruga ostao isti, a time i struje i snage.

V. $S_{uk}' = S_{uk} + Q_{C_k} = 0.4121 + 0.1869 \cdot i - 3142.25^2 \cdot C_k \cdot i =$

$= 0.4121 + \underbrace{(0.1869 - 1963750 \cdot C_k)}_{=0} \cdot i$

VI.

$\Rightarrow 1963750 \cdot C_k = 0.1869 \rightarrow C_k = 9.5175 \cdot 10^{-8}$ F

$\Rightarrow \boxed{C_k = 95.175 \text{ nF}}$

VII. $F.S. = \frac{P}{|S|} = \cos(\varphi_s)$; ako $P = |S|$ onda $F.S. = 1$

VIII. $F.S. = \frac{P}{|S|} = \frac{0.4121}{0.4525} = \boxed{91.1\ \%}$

c) I. $C_k' = C_k - 20\% \cdot C_k = 0.8 \cdot C_k = \boxed{76.14\ \mu F}$

II. $Q_{C_k}' = \omega \cdot C_k' \cdot |U|^2 \angle -90° = \boxed{-0.1495\ VAr}$

(vidi b) III.)

III. $S_{uk}' = S_{uk} + Q_{C_k}' = 0.4121 + 0.1869 \cdot i - 0.1495 i =$

$= 0.4121 + 0.0374 i = 0.4138 \angle 5.2°\ VA$

$F.S.' = \frac{0.4121}{0.4138} = \boxed{99.6\ \%}$

IV. Budući da razlika u iznosu kapaciteta od 20% uzrokuje smanjenje faktora snage od manje od 0.5%, čini se da nije prevažan egzaktan iznos kompenzirajućeg kondenzatora, dok god je istog reda veličine.

d) I. Imaginarni dio ukupne impedancije mora biti 0.

II. Ukupna impedancija ovisi o frekvenciji izvora, zbog Z_L i Z_C.

III. $Z_{uk} = Z_L + Z_{R_1} + \frac{Z_{R_2} \cdot Z_C}{Z_{R_2} + Z_C} = \omega \cdot L \cdot i + R_1 + \frac{R_2 \cdot \left(\frac{-i}{\omega C}\right)}{R_2 - \frac{i}{\omega \cdot C}} =$

$$= R_1 + \omega \cdot L \cdot i + \frac{-i \cdot R_2}{\omega \cdot C} \cdot \frac{\left(R_2 + \frac{i}{\omega \cdot C}\right)}{\left(R_2 - \frac{i}{\omega \cdot C}\right)\left(R_2 + \frac{i}{\omega \cdot C}\right)} =$$

$$= R_1 + \omega \cdot L \cdot i + \frac{\left(\frac{-i \cdot R_2^2}{\omega \cdot C} - \frac{\overbrace{i^2}^{=-1} R_2}{\omega^2 \cdot C^2}\right)}{\left(R_2^2 + \frac{1}{\omega^2 C^2}\right)} =$$

$$= \underbrace{R_1 + \frac{\left(\frac{R_2}{\omega^2 C^2}\right)}{\left(R_2^2 + \frac{1}{\omega^2 C^2}\right)}}_{Re} + \underbrace{\omega \cdot L \cdot i - \frac{\left(\frac{R_2^2}{\omega \cdot C}\right)}{\left(R_2^2 + \frac{1}{\omega^2 C^2}\right)} \cdot i}_{Im}$$

IV.

$$Im(Z_{ik}) = 0 \Rightarrow \omega_{Kz} \cdot L - \frac{\left(\frac{R_2^2}{\omega_{Kz} \cdot C}\right)}{\left(R_2^2 + \frac{1}{\omega_{Kz}^2 C^2}\right)} = 0$$

$$\Rightarrow \omega_{Kz} \cdot L = \frac{\left(\frac{R_2^2}{\omega_{Kz} \cdot C}\right)}{\left(R_2^2 + \frac{1}{\omega_{Kz}^2 \cdot C^2}\right)} \Big/ \cdot \left(R_2^2 + \frac{1}{\omega_{Kz}^2 C^2}\right) \cdot \omega_{Kz} \cdot C$$

$$\Rightarrow \omega_{Kz}^2 \cdot L \cdot C \cdot R_2^2 + \frac{L}{C} = R_2^2$$

$$\Rightarrow \omega_{Kz}^2 = \frac{R_2^2 - \frac{L}{C}}{L \cdot C \cdot R_2^2} = \frac{1}{L \cdot C} - \frac{1}{R_2^2 \cdot C^2}$$

$$\Rightarrow \omega_{rez} = \sqrt{\frac{1}{L\cdot C} - \frac{1}{R_2^2\cdot C^2}} = \sqrt{\frac{1}{0.42\cdot 210\cdot 10^{-9}} - \frac{1}{1500^2\cdot 210^2\cdot 10^{-18}}} =$$

$$= \sqrt{\frac{10\cdot 10^8}{0.42\cdot 210} - \frac{10^8}{1.5^2\cdot 2.1^2}} = 0.11224\cdot 10^4 \ \frac{rad}{s} =$$

$$\boxed{= 1122.4 \ \frac{rad}{s}}$$

V. $\omega_{rez} = 2\cdot\pi\cdot f_{rez} \Rightarrow \boxed{f_{rez} = \frac{\omega_{rez}}{2\cdot\pi} = 178.6 \ Hz}$

VI. $\omega_{rez} = \sqrt{\frac{1}{L\cdot C} - \frac{1}{R_2^2\cdot C^2}} = \sqrt{-0.79365\cdot 10^4} = \underline{\underline{??!}}$

Frekvencija ne može biti imaginarna! To nema smisla, nije fizikalno, ne postoji ...

U postupku u IV. dijelili smo jednadžbu s ω_{rez}, a to se smije samo ako je $\omega_{rez} \neq 0$.

Za $\omega = 0$ vidimo da $Z_{uk} = R_1 + R_2$, dakle isto je $Im(Z_{uk}) = 0$. U prijevodu, ne postoji AC napon pri kojem bi takav krug rezonirao.